AP Calculus AB
2023 Summer Work

Welcome to AP Calculus AB!

Please take your time to go through this packet throughout the summer. You are encouraged to work with a friend to complete the packet. These concepts are important to understand prior to the start of Calculus. These concepts are the most important concepts pulled from your Algebra 2 and Pre-Calculus courses, which are crucial to succeed in Calculus. Try to complete this packet without the use of a calculator, unless otherwise stated. I look forward to working with each of you in the Fall! Please reach out if you have any questions.

Enjoy your summer!!

Mrs. Marsh
lmarsh@town.hull.ma.us

Given $f(x) = x^2 - 2x + 5$, find the following.

1. $f(-2) =$

2. $f(x + 2) =$

3. $f(x + h) =$

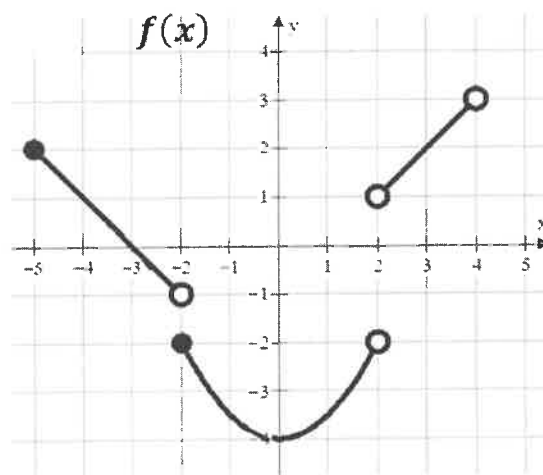**4. Use the graph $f(x)$ to answer the following.**

$f(0) =$ $f(4) =$

$f(-1) =$ $f(-2) =$

$f(2) =$ $f(3) =$

$f(x)$

$f(x) = 2$ when $x =$?

$f(x) = -3$ when $x =$?

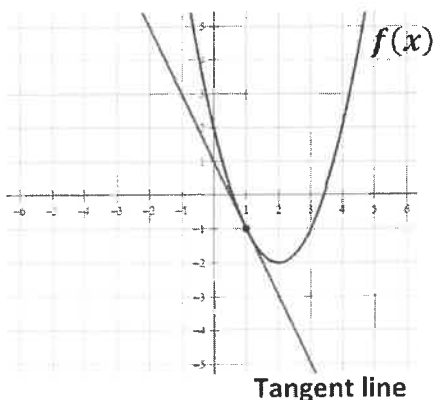**Write the equation of the line for the following. Use point-slope form.** $y - y_1 = m(x - x_1)$

5. slope $= 3$ and $(4, -2)$

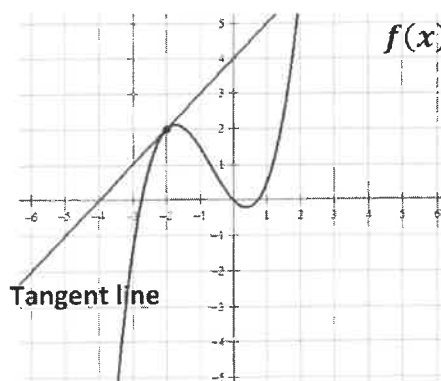6. slope $= -\frac{3}{2}$ and $f(-5) = 7$

7. $f(4) = -8$ and $f(-3) = 12$

Write the equation of the tangent line in point-slope form. $y - y_1 = m(x - x_1)$
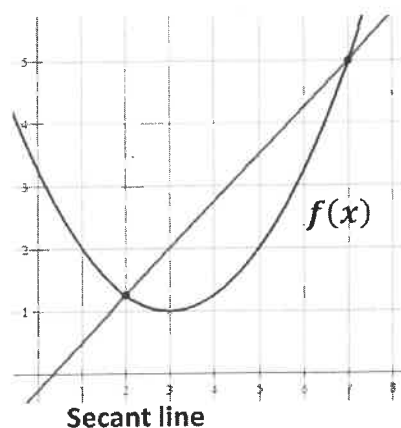
8. The line tangent to $f(x)$ at $x = 1$

Tangent line

9. The line tangent to $f(x)$ at $x = -2$

Tangent line

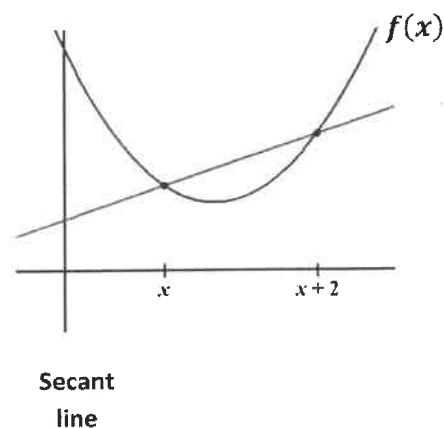**Multiple Choice. Remember slope** $= \dfrac{y_2 - y_1}{x_2 - x_1}$

10. Which choice represents the slope of the secant line shown?

A) $\dfrac{7-2}{f(7)-f(2)}$ B) $\dfrac{f(7)-2}{7-f(2)}$ C) $\dfrac{7-f(2)}{f(7)-2}$ D) $\dfrac{f(7)-f(2)}{7-2}$
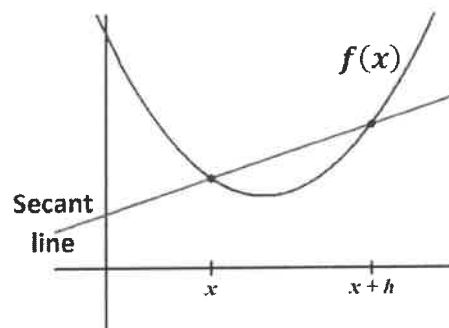

Secant line

11. Which choice represents the slope of the secant line shown?

A) $\dfrac{f(x)-f(x+2)}{x+2-x}$ B) $\dfrac{f(x+2)-f(x)}{x+2-x}$

C) $\dfrac{f(x+2)-f(x)}{x-(x+2)}$ D) $\dfrac{x+2-x}{f(x)-f(x+2)}$

Secant line

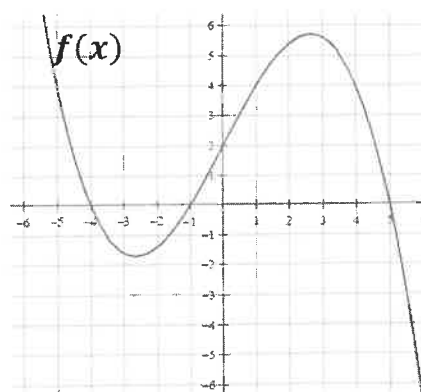12. Which choice represents the slope of the secant line shown?

A) $\dfrac{f(x+h)-f(x)}{x-(x+h)}$ B) $\dfrac{x-(x+h)}{f(x+h)-f(x)}$ C) $\dfrac{f(x+h)-f(x)}{x+h-x}$

D) $\dfrac{f(x)-f(x+h)}{x+h-x}$

13. Which of the following statements about the function *f(x)* is true?

    I. $f(2) = 0$
    II. $(x + 4)$ is a factor of $f(x)$
    III. $f(5) = f(-1)$

A) I only
B) II only
C) III only
D) I and III only
E) II and III only

Find the domain and range (express in interval notation). Find all horizontal and vertical asymptotes.

14.
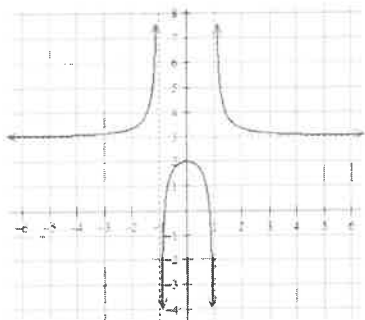
Domain:

Range:

Horizontal Asymptote(s):

Vertical Asymptotes(s):

15.

Domain:

Range:

Horizontal Asymptote(s):

Vertical Asymptotes(s):

16.

Domain:

Range:

Horizontal Asymptote(s):

Vertical Asymptotes(s):

Multiple Choice

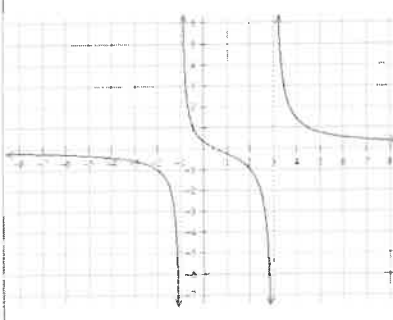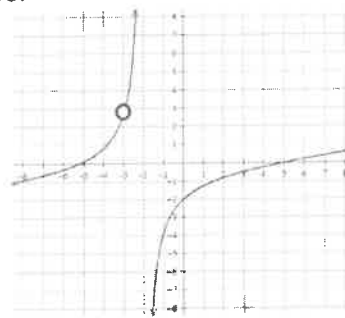17. Which of the following functions has a vertical asymptote at $x = 4$?

   (A) $\dfrac{x+5}{x^2-4}$

   (B) $\dfrac{x^2-16}{x-4}$

   (C) $\dfrac{4x}{x+1}$

   (D) $\dfrac{x+6}{x^2-7x+12}$

   (E) None of the above

18. Consider the function: $(x) = \dfrac{x^2-5x+6}{x^2-4}$. Which of the following statements is true?

   I. $f(x)$ has a vertical asymptote of $x = 2$
   II. $f(x)$ has a vertical asymptote of $x = -2$
   III. $f(x)$ has a horizontal asymptote of $y = 1$

   (A) I only

   (B) II only

   (C) I and III only

   (D) II and III only

   (E) I, II and III

Rewrite the following using rational exponents. Example: $\dfrac{1}{\sqrt[3]{x^2}} = x^{-\frac{2}{3}}$

19. $\sqrt[5]{x^3} + \sqrt[5]{2x}$	20. $\sqrt{x+1}$	21. $\dfrac{1}{\sqrt{x+1}}$
22. $\dfrac{1}{\sqrt{x}} - \dfrac{2}{x}$	23. $\dfrac{1}{4x^3} + \dfrac{1}{2}\sqrt[4]{x^3}$	24. $\dfrac{1}{4\sqrt{x}} - 2\sqrt{x+1}$

Write each expression in radical form and positive exponents. Example: $x^{-\frac{2}{3}} + x^{-2} = \dfrac{1}{\sqrt[3]{x^2}} + \dfrac{1}{x^2}$

25. $x^{-\frac{1}{2}} - x^{\frac{3}{2}}$	26. $\dfrac{1}{2}x^{-\frac{1}{2}} + x^{-1}$	27. $3x^{-\frac{1}{2}}$
28. $(x+4)^{-\frac{1}{2}}$	29. $x^{-2} + x^{\frac{1}{2}}$	30. $2x^{-2} + \dfrac{3}{2}x^{-1}$

Need to know basic trig functions in RADIANS! We _never_ use degrees. You can either use the Unit Circle or Special Triangles to find the following.

31. $\sin\frac{\pi}{6}$	32. $\cos\frac{\pi}{4}$	33. $\sin 2\pi$
34. $\tan\pi$	35. $\sec\frac{\pi}{2}$	36. $\cos\frac{\pi}{6}$
37. $\sin\frac{\pi}{3}$	38. $\sin\frac{3\pi}{2}$	39. $\tan\frac{\pi}{4}$
40. $\csc\frac{\pi}{2}$	41. $\sin\pi$	42. $\cos\frac{\pi}{3}$
43. Find x where $0 \le x \le 2\pi$, $$\sin x = \frac{1}{2}$$	44. Find x where $0 \le x \le 2\pi$, $$\tan x = 0$$	45. Find x where $0 \le x \le 2\pi$, $$\cos x = -1$$

Solve the following equations. Remember $e^0 = 1$ and $\ln 1 = 0$.

46. $e^x + 1 = 2$	47. $3e^x + 5 = 8$	48. $e^{2x} = 1$
49. $\ln x = 0$	50. $3 - \ln x = 3$	51. $\ln(3x) = 0$
52. $x^2 - 3x = 0$	53. $e^x + xe^x = 0$	54. $e^{2x} - e^x = 0$

Solve the following trig equations where $0 \leq x \leq 2\pi$.

55. $\sin x = \frac{1}{2}$	56. $\cos x = -1$	57. $\cos x = \frac{\sqrt{3}}{2}$
58. $2\sin x = -1$	59. $\cos x = \frac{\sqrt{2}}{2}$	60. $\cos\left(\frac{x}{2}\right) = \frac{\sqrt{3}}{2}$
61. $\tan x = 0$	62. $\sin(2x) = 1$	63. $\sin\left(\frac{x}{4}\right) = \frac{\sqrt{3}}{2}$

For each function, determine its domain and range.

Function	Domain	Range
64. $y = \sqrt{x-4}$		
65. $y = (x-3)^2$		
66. $y = \ln x$		
67. $y = e^x$		
68. $y = \sqrt{4-x^2}$		

Simplify.

69. $\dfrac{\sqrt{x}}{x}$

70. $e^{\ln x}$

71. $e^{1+\ln x}$

72. $\ln 1$

73. $\ln e^7$

74. $\log_3 \frac{1}{3}$

75. $\log_{1/2} 8$

76. $\ln \frac{1}{2}$

(calculator OK)

77. $27^{\frac{2}{3}}$

78. $\left(5a^{2/3}\right)\left(4a^{3/2}\right)$

79. $\dfrac{4xy^{-2}}{12x^{-\frac{1}{3}}y^{-5}}$

80. $\left(4a^{5/3}\right)^{3/2}$

If $f(x) = \{(3,5),\ (2,4),\ (1,7)\}$, $g(x) = \sqrt{x-3}$
$h(x) = \{(3,2),\ (4,3),\ (1,6)\}$, $k(x) = x^2 + 5$, then determine each of the following.

81. $(f+h)(1)$

82. $(k-g)(5)$

83. $f\big(h(3)\big)$

84. $g\big(k(7)\big)$

85. $h(3)$

86. $g\big(g(9)\big)$

87. $f^{-1}(4)$

88. $k^{-1}(x)$

89. $k\big(g(x)\big)$

90. $g\big(f(2)\big)$