Pre-Calculus Summer Packet
Mr. Russell

Congratulations! You made it to Pre-Calculus. I am Mr. Russell and I'm very excited to be a part of your academic growth. Pre-Calculus is supposed to bridge the gap between Algebra and Calculus, so hopefully you are taking this class to pursue a Calculus class in the future. If not, we will still get the best out of it. This packet is designed to keep your math skills fresh, so ideally you want to do a little here and there throughout the summer (not waiting until the last week of August). This packet will be graded (at least a double homework assignment). Please have your work on a separate piece of paper. It will be graded on completion but most importantly effort. I already started a google classroom so if you have questions on this packet you can reach out to me (I will do the best I can to get back to you quickly). Enjoy the summer!

"Your success will be determined by your own confidence and fortitude."

Michelle Obama AUTHOR, FORMER FIRST LADY

AP Pre-Calculus Google Classroom:osj2i4v

Honors Pre-Calculus Google Classroom::5no2ami

Radicals:

To simplify means that 1) no radicand has a perfect square factor and

2) there is no radical in the denominator (rationalize).

Recall the **Product Property** $\sqrt{ab}=\sqrt{a}\bullet\sqrt{b}$ and the **Quotient Property** $\sqrt{\frac{a}{b}}=\frac{\sqrt{a}}{\sqrt{b}}$

Examples: Simplify $\sqrt{24}=\sqrt{4}\bullet\sqrt{6}$ find the perfect square factor

$=2\sqrt{6}$ simplify

Simplify $\sqrt{\frac{7}{2}}=\frac{\sqrt{7}}{\sqrt{2}}\bullet\frac{\sqrt{2}}{\sqrt{2}}$ multiply numerator & denominator by $\sqrt{2}$

$=\frac{\sqrt{14}}{\sqrt{4}}=\frac{\sqrt{14}}{2}$ multiply straight across and simplify

If the denominator contains 2 terms, multiply the numerator and denominator by conjugate of the denominator (the conjugate of $3+\sqrt{2}$ is $3-\sqrt{2}$)

Simplify

1) $\frac{5\sqrt{9}}{\sqrt{16}}$

2) $\frac{2\sqrt{3}}{\sqrt{16}}$

3) $\frac{5\sqrt{2}}{\sqrt{8}}$

4) $\frac{4\sqrt{6}}{2\sqrt{27}}$

5) $\frac{3\sqrt{15}}{2\sqrt{80}}$

6) $\frac{2\sqrt{4}}{2\sqrt{16}}$

7. $\frac{5}{-3-3\sqrt{3}}$

8. $\frac{2+5\sqrt{3}}{-4+4\sqrt{2}}$

Complex Numbers:

Form of complex number: $a+bi$

Where a is the real part and the bi is the imaginary part

Always make these substitutions $\sqrt{-1}=i$ and $i^2=-1$

To simplify: pull out the $\sqrt{-1}$ before performing any operation

Example: $\sqrt{-5}=\sqrt{-1}\bullet\sqrt{5}$ Pull out $\sqrt{-1}$ Example: $(i\sqrt{5})^2=i\sqrt{5}\bullet i\sqrt{5}$

$=i\sqrt{5}$ Make substitution $=i^2\sqrt{25}=(-1)(5)=-5$

Treat i like any other variable when $+,-,\times,$ or $\div$ (but always simplify $i^2=-1$)

Example:	$2i(3+i)=2(3i)+2i(i)$	Distribute
	$=6i+2i^2$	Simplify
	$=6i+2(-1)$	Substitute
	$=-2+6i$	Simplify and rewrite in complex form

Since $i=\sqrt{-1}$, no answer can have an 'i' in the denominator. RATIONALIZE!

1. $(2-4i)(-6+4i)$

2. $(-3+2i)(-6-8i)$

3. $(1-7i)^2$

4. $6(-7+6i)(-4+2i)$

5. $(-2-2i)(-4-3i)(7+8i)$

6. $5i+7i\cdot i$

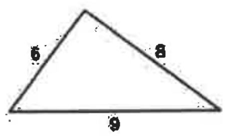
7. $-3i\cdot 6i-3(-7+6i)$

8. $-6i(8-6i)(-8-8i)$

Pythagorean Theorem (right triangles): $a^2 + b^2 = c^2$

Do the following lengths form a right triangle? (does $a^2 + b^2 = c^2$)

1)



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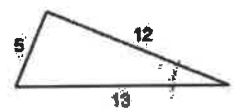

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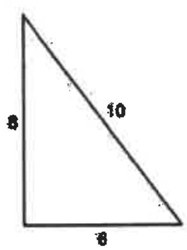

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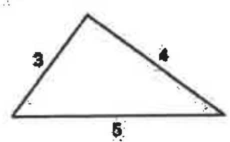
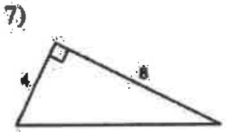


5) $a = 6.4,\ b = 12,\ c = 12.2$

6) $a = 2.1,\ b = 7.2,\ c = 7.5$

Find each missing length to the nearest tenth.

7)



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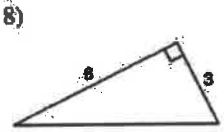

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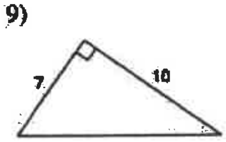

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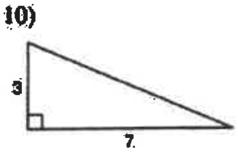


Equations of Lines:	
Slope-intercept form: $y = mx + b$	Vertical line: $x = c$ *(slope is undefined)*
Point-slope form: $y - y_1 = m(x - x_1)$	Horizontal line: $y = c$ *(slope is zero)*
Standard Form: $Ax + By = C$	Slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$

Write the slope-intercept form of the equation of each line

1) $3x - 2y = -16$

2) $13x - 11y = -12$

3) $9x - 7y = -7$

4) $x - 3y = 6$

5) $6x + 5y = -15$

6) $4x - y = 1$

Write the standard form of the equation of the line through the given point with the given slope.

7. through: (1, 2), slope = 7

8. through: (3, −1), slope = −1

9. through: (2, 5), slope = undefined

Write the point-slope form of the equation of the line described

10.. through: (4, 2), parallel to $y = -\frac{3}{4}x - 5$

11. through: (−1, 4), perpendicular to $y = -5x + 2$

Systems of Equations:

$$\begin{cases} 3x+y=6 \\ 2x-2y=4 \end{cases}$$

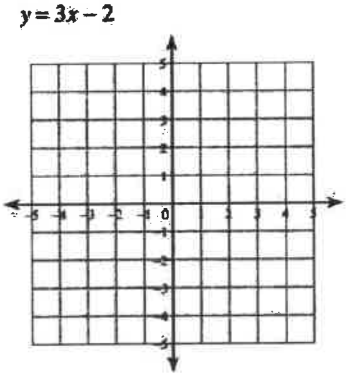
Substitution:	**Elimination:**
Solve 1 equation for 1 variable	Find opposite coefficients for 1 variable
Rearrange.	Multiply equation(s) by constant(s).
Plug into 2nd equation.	Add equations together (lose 1 variable)
Solve for the other variable.	Solve for variable.

Then plug answer back into an original equation to solve for the 2nd variable.

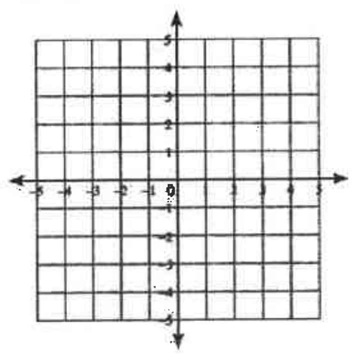
$y=6-3x$	Solve 1st equation for y	$6x+2y=12$	Multiply 1st equation by 2
$2x-2(6-3x)=4$	Plug into 2nd equation	$\underline{2x-2y=4}$	coefficients of y are opposite
$2x-12+6x=4$	Distribute	$8x=16$	Add
$8x=16$ and $x=2$	Simplify	$x=2$	Simplify.

Solve each system by graphing. (Where do they intersect).

1) $y=-3x+4$
$y=3x-2$



2) $y=x+2$
$x=-3$



Solve each system by substitution.

3. y = 4x − 9
y = x − 3

4. 4x + 2y = 10
x − y = 13

Solve each system by elimination

5. $8x - 6y = -20$
   $-16x + 7y = 30$

6. $6x - 12y = 24$
   $-x - 6y = 4$

Exponents:

Recall the following rules of exponents:

1. $a^1 = a$ — Any number raised to the power of one equals itself.
2. $1^n = 1$ — One raised to any power is one.
3. $a^0 = 1$ — Any nonzero number raised to the power of zero is one.
4. $a^m \cdot a^n = a^{m+n}$ — When multiplying two powers that have the same base, add the exponents.
5. $\dfrac{a^m}{a^n} = a^{m-n}$ — When dividing two powers with the same base, subtract the exponents.
6. $(a^m)^n = a^{mn}$ — When a power is raised to another power, multiply the exponents.
7. $a^{-n} = \dfrac{1}{a^n}$ and $\dfrac{1}{a^{-n}} = a^n$ — Any nonzero number raised to a negative power equals its reciprocal raised to the opposite positive power.

Simplify each of the following.

1) $a \cdot a^2 \cdot a^3$ 2) $(2a^2b)(4ab^2)$ 3) $(6x^2)(-3x^5)$ 4) $b^3 \cdot b^4 \cdot b^7 \cdot b$ 5) $(3x^3)(3x^4)(-3x^2)$

6) $(2x^2y^3)^2$ 7) $(5x^2y^4)^3$ 8) $(6x^4y^6)^3$ 9) $(4x^3y^3)^3$ 10) $(7xy)^2$

11) $\dfrac{x^3}{x}$ 12) $\dfrac{18c^3}{-3c^2}$ 13) $\dfrac{9a^3b^5}{-3ab^2}$ 14) $\dfrac{-48c^2d^4}{-8cd}$ 15) $\dfrac{22y^6z^8}{2yz^{-7}}$

Polynomials:

To add/subtract polynomials, combine like terms.

EX:	$8x-3y+6-(6y+4x-9)$	*Distribute the negative through the parantheses.*
	$=8x-3y+6-6y-4x+9$	*Combine like terms with similar variables.*
	$=8x-4x-3y-6y+6+9$	
	$=4x-9y+15$	

Add the following polynomials

1-3

$(7j^3-2)+(5j^3-j-3)$
$(8a^5-4)+(3a^5+a-2)$
$(6m^5+1)+(2m^5+9m-1)$

Subtract the following polynomials

4-6

$(-x^2+x-4)-(3x^2-8x-2)$
$(8x^2-3x)-(5x-5-8x^2)$
$(-x^2-5x-3)-(-7x^2-8x-8)$

Multiply the following polynomials

7-9

$(8x^3y^2)(-3x^2y^3)$
$(-9x^3y)(-8x^2y^3)$
$j^2(k^5j^3)$

Divide the following polynomials

10-12

$\frac{9x-6}{3}$

$\frac{36x^2-72x}{9x^3}$

To multiply two binomials, use FOIL.

EX: $(3x-2)(x+4)$ — *Multiply the first, outer, inner, and last terms.*

$=3x^2+12x-2x-8$ — *Combine like terms together.*

$=3x^2+10x-8$

Factoring:

Follow these steps in order to factor polynomials.

STEP 1: **Look for a GCF in ALL of the terms.**

a) **If you have one (other than 1) factor it out.**

b) **If you don't have one move on to STEP 2**

STEP 2: How many terms does the polynomial have?

2 Terms **a) Is it the difference of two squares?** $a^2-b^2=(a+b)(a-b)$

EX: $x^2-25=(x+5)(x-5)$

b) Is it the sum or difference of two cubes? $a^3-b^3=(a-b)(a^2+ab+b^2)$
$a^3+b^3=(a+b)(a^2-ab+b^2)$

EX: $m^3+64=(m+4)(m^2-4m+16)$
$p^3-125=(p-5)(p^2+5p+25)$

3 Terms	**EX:**
$x^2+bx+c=(x+\_)(x+\_)$	$x^2+7x+12=(x+3)(x+4)$
$x^2-bx-c=(x-\_)(x-\_)$	$x^2-5x+4=(x-1)(x-4)$
$x^2+bx-c=(x-\_)(x+\_)$	$x^2+6x-16=(x-2)(x+8)$
$x^2-bx-c=(x-\_)(x+\_)$	$x^2-2x-24=(x-6)(x+4)$

4 Terms—Factor by Grouping

a) **Pair up first two terms and last two terms.**

b) **Factor out GCF of each pair of numbers.**

c) **Factor out front parentheses that the terms have in common.**

d) **Put leftover terms in parentheses.**

$$Ex: x^3+3x^2+9x+27=\left(x^3+3x^2\right)+\left(9x+27\right)$$
$$=x^2(x+3)+9(x+3)$$
$$=(x+3)\left(x^2+9\right)$$

Factor the Following

1-6

$y^3 + 9y^2$

$5x^2y^3 + 15x^3y^2$

$12t^5 - 20t^4 + 8t^2 - 16$

$p^2 - 36$

$25 - x^2$

$4a^3 - 49a$

7-11

$x^2 - 6x - 16$

$x^2 - 10xy + 24y^2$

$x^2 + 3x + 2$

$x^2 - 3x + 2$

$x^2 - x - 30$

To solve quadratic equations, try to factor first and set each factor equal to zero. Solve for your variable. If the quadratic does NOT factor, use the quadratic formula.

EX: $x^2 - 4x = 21$ *Set equal to zero FIRST.*

$x^2 - 4x - 21 = 0$ *Now factor.*

$(x+3)(x-7) = 0$ *Set each factor equal to zero.*

$x+3=0 \quad x-7=0$ *Solve for each x.*

$x=-3 \quad x=7$

Solve for x

1-3

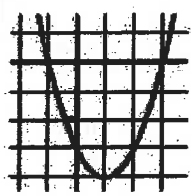
$3x^2 - 27x + 42 = 0$

$4x^2 + 20x + 25 = 0$

$3y^2 + 4y - 4 = 0$

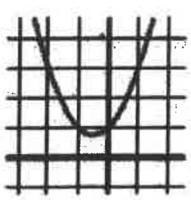
Discriminant: The number under the radical in the quadratic formula $(b^2 - 4ac)$ can tell you what kind of roots you will have.

If $b^2 - 4ac > 0$ you will have TWO real roots (touches the x-axis twice)



If $b^2 - 4ac = 0$ you will have ONE real root (touches axis once)

If $b^2 - 4ac < 0$ you will have TWO imaginary roots. (Function does not cross the x-axis)



QUADRATIC FORMULA—allows you to solve any quadratic for all its real and imaginary roots.

$5x^2 - 2x + 4 = 0 \quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

EX: In the equation $x^2+2x+3=0$, find the value of the discriminant, describe the nature of the roots, then solve.

$x^2+2x+3=0$ *Determine the values of a, b, and c.*

$a=1\ b=2\ c=3$ *Find the discriminant.*

$D=2^2-4\cdot1\cdot3$

$D=4-12$

$D=-8$ *There are two imaginary roots.*

$$Solve:\ x=\frac{-2\pm\sqrt{-8}}{2}$$

$$x=\frac{-2\pm2i\sqrt{2}}{2}$$

$$x=-1\pm i\sqrt{2}$$

Find the value of the discriminant of each quadratic equation.

1) $6p^2-2p-3=0$

2) $-2x^2-x-1=0$

3) $-4m^2-4m+5=0$

4) $5b^2+b-2=0$

Solve each equation with the quadratic formula.

1) $m^2-5m-14=0$

2) $b^2-4b+4=0$

Composition and Inverses of Functions:

Recall: $(f\ \ g)(x)=f(g(x))$ OR $f[g(x)]$ read "f of g of x" means to plug the inside function in for x in the outside function.

Example: Given $f(x)=2x^2+1$ *and* $g(x)=x-4$ find $f(g(x))$.

$$f(g(x))=f(x-4)$$
$$=2(x-4)^2+1$$
$$=2(x^2-8x+16)+1$$
$$=2x^2-16x+32+1$$
$$f(g(x))=2x^2-16x+33$$

For 1- 9: Let $f(x) = 2x - 1$, $g(x) = 3x$, and $h(x) = x^2 + 1$. Compute the following:

1. $f(g(-3))$ **2.** $f(h(7))$ **3.** $(g \circ h)(24)$

4. $f(g(h(2)))$ **5.** $h(g(f(5)))$ **6.** $g(f(h(-6)))$

To find the inverse of a function, simply switch the x and the y and solve for the new "y" value.

Summer Review Packet for Students Entering Pre-Calculus

Example:	$f(x) = \sqrt[3]{x+1}$	**Rewrite *f(x)* as *y***
	$y = \sqrt[3]{x+1}$	**Switch *x* and *y***
	$x = \sqrt[3]{y+1}$	**Solve for your new *y***
	$(x)^3 = \left(\sqrt[3]{y+1}\right)^3$	**Cube both sides**
	$x^3 = y+1$	**Simplify**
	$y = x^3 - 1$	**Solve for *y***
	$f^{-1}(x) = x^3 - 1$	**Rewrite in inverse notation**

Find the inverse, $f^{-1}(x)$, if possible.

Note: Switch x and y and 1 and 2.

Inverse Relations

Find the inverse for each relation.

1. $\{(1, -3), (-2, 3), (5, 1), (6, 4)\}$ 2. $\{(-5, 7), (-6, -8), (1, -2), (10, 3)\}$

Finding Inverses

Find an equation for the inverse for each of the following relations.

3. $y = 3x + 2$ 4. $y = -5x - 7$ 5. $y = 12x - 3$

6. $y = -8x + 16$ 7. $y = \frac{2}{3}x - 5$ 8. $y = -\frac{3}{4}x + 5$

Rational Algebraic Expressions

Multiplying and Dividing: **Factor numerator and denominator completely. Cancel any common factors in the top and bottom. If dividing, change divide to multiply and flip the second fraction.**

EX: $\dfrac{x^2+10x+21}{5-4x-x^2}\bullet\dfrac{x^2+2x-15}{x^3+4x^2-21x}$ *Factor everything completely.*

$=\dfrac{(x+7)(x+3)}{(5+x)(1-x)}\bullet\dfrac{(x+5)(x-3)}{x(x-3)(x+7)}$ ***Cancel out common factors in the top and bottom.***

$=\dfrac{(x+3)}{x(1-x)}$ ***Simplify.***

First find the least common denominator. Write each fraction with that LCD. Add/subtract numerators as indicated and leave the denominators as they are.

EX: $\dfrac{3x+1}{x^2+2x}+\dfrac{5x-4}{2x+4}$ *Factor denominator completely.*

$\dfrac{3x+1}{x(x+2)}+\dfrac{5x-4}{2(x+2)}$ *Find LCD, which is $(2x)(x+2)$*

$\dfrac{2(3x+1)}{2x(x+2)}+\dfrac{x(5x-4)}{2x(x+2)}$ *Rewrite each fraction with the LCD in the denominator.*

$\dfrac{6x+2+5x^2-4x}{2x(x+2)}$ *Write as one fraction.*

$\dfrac{5x^2+2x+2}{2x(x+2)}$ *Combine like terms.*

Solving Rational Equations:

Multiply each term by the LCD of all the fractions. This should eliminate all of our fractions. Then solve the equation as usual.

$\dfrac{5}{x+2}+\dfrac{1}{x}=\dfrac{5}{x}$ *Find LCD first* $x(x+2)$

$x(x+2)\dfrac{5}{x+2}+x(x+2)\dfrac{1}{x}=\dfrac{5}{x}x(x+2)$ *Multiply each term by the LCD.*

$5x+1(x+2)=5(x+2)$ *Simplify and solve.*

$5x+x+2=5x+10$

$6x+2=5x+10$

$x=8$ ← *Check your answer! Sometimes they do not check!*

Check: $\dfrac{5}{8+2}+\dfrac{1}{8}=\dfrac{5}{8}$

$\dfrac{5}{10}+\dfrac{1}{8}=\dfrac{5}{8}$

$\dfrac{5}{8}=\dfrac{5}{8}$

Simplify and state extraneous solutions (if any).

1-10

$$\frac{p+4}{p^2+6p+8}$$

$$\frac{a^2+5a+4}{a^2+9a+20}$$

$$\frac{(v-7)(v+8)}{(v+8)(v-10)} \div \frac{1}{v-10}$$

$$\frac{x-8}{(x+6)(x-8)} \cdot \frac{4x(x+10)}{x+10}$$

$$\frac{70v^2}{100v}$$

$$\frac{u-v}{8v} + \frac{6u-3v}{8v}$$

$$\frac{3}{4v^2+4v} - \frac{7}{2}$$

$$6 - \frac{x+5}{(7x-5)(x+4)}$$

$$1 = \frac{1}{x^2+2x} + \frac{x-1}{x}$$

$$\frac{5}{n^3+5n^2} = \frac{4}{n+5} + \frac{1}{n^2}$$