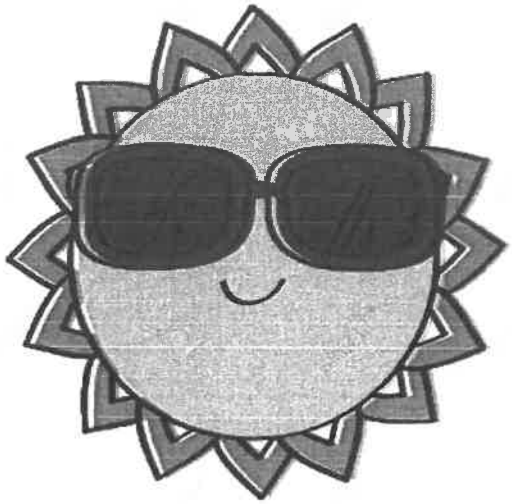


Summer Math Packet

Students entering 8th grade
Fall of 2021

Name: ______________________________

Date Completed: __________

Hello! I hope you enjoy your summer, but it is good to work throughout the summer to keep your skills sharp. This packet may seem long, but most of the problems should be quick to solve and there are some diagrams that take up space. Please do not use a calculator and I look forward to seeing you in the fall.

Sincerely,

Mrs. Cocchi
MMS Math Department
jcocchi@town.hull.ma.us

Multiplying

In arithmetic we wrote multiplication problems these two ways:

$$3 \times 5 = 15 \qquad \begin{array}{r} 3 \\ \times\, 5 \\ \hline 15 \end{array}$$

In algebra we show multiplication by using a **dot** or by using **parentheses**. Below are some examples.

$$3 \cdot 5 = 15 \qquad 3(5) = 15 \qquad (3)(5) = 15$$

Here are some multiplication problems for you to do:

$4 \cdot 3 =$	$10 \cdot 7 =$	$2 \cdot 4 =$
$3 \cdot 6 =$	$(8)(3) =$	$(11)(2) =$
$(8)(4) =$	$3 \cdot 5 \cdot 5 =$	$8(9) =$
$4(4) =$	$3(9) =$	$10 \cdot 10 =$
$9 \cdot 10 =$	$1 \cdot 5 =$	$6 \cdot 5 =$
$6 \cdot 6 =$	$4 \cdot 9 =$	$9 \cdot 9 =$

Below is a multiplication table that needs to be finished. Finish it and then use it to check the problems you just did.

	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6						
2	2	4	6	8								
3	3	6	9									
4												
5												
6								48				
7												
8												
9												
10												
11												
12												

63	56	28
81	49	100
77	39	132
30	30	30
48	48	48
17	5	23

rime Factors

ere are some numbers that we have already broken down:

24	14	64
12 · 2	2 · 7	8 · 8

owever, only one of them has been broken down all the way. This time we are going to 'eak them all the way down into **prime factors**:

24
12 · 2
3 · 4 · 2
3 · 2 · 2 · 2

$24 = 3 \cdot 2 \cdot 2 \cdot 2$

14
2 · 7

$14 = 2 \cdot 7$

64
8 · 8
4 · 2 2 · 4
2 · 2 2 · 2

$64 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

eak down each number into prime factors.

70

70 =

12

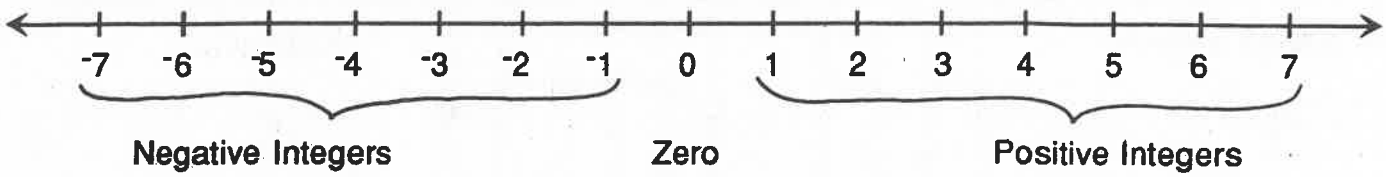
12 =

22

22 =

Integers

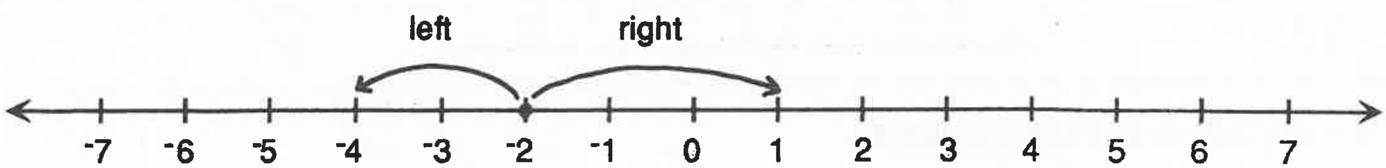
Integers are a lot like the whole numbers that you already know. The main difference is that there are **negative integers** as well as **positive integers**. **Zero** is also an integer. Here is one way we can picture the set of integers:



As you can see, the negative integers are to the left of zero. We use a little raised minus sign to show that an integer is negative. Sometimes we use a little raised plus sign to show that an integer is positive, but we usually don't use any sign at all when the integer is positive. Zero is neither positive nor negative.

Comparing Integers

By looking at the number line we can easily tell which integers are **greater** (larger) than a certain number and which are **less** (smaller) than the number.



1 is greater than $^-2$ because it is to the *right* of $^-2$.

$^-4$ is less than $^-2$ because it is to the *left* of $^-2$.

Write >, = or < between the two integers to show whether the first is greater than, equal to or less than the second.

1 is greater than $^-2$.

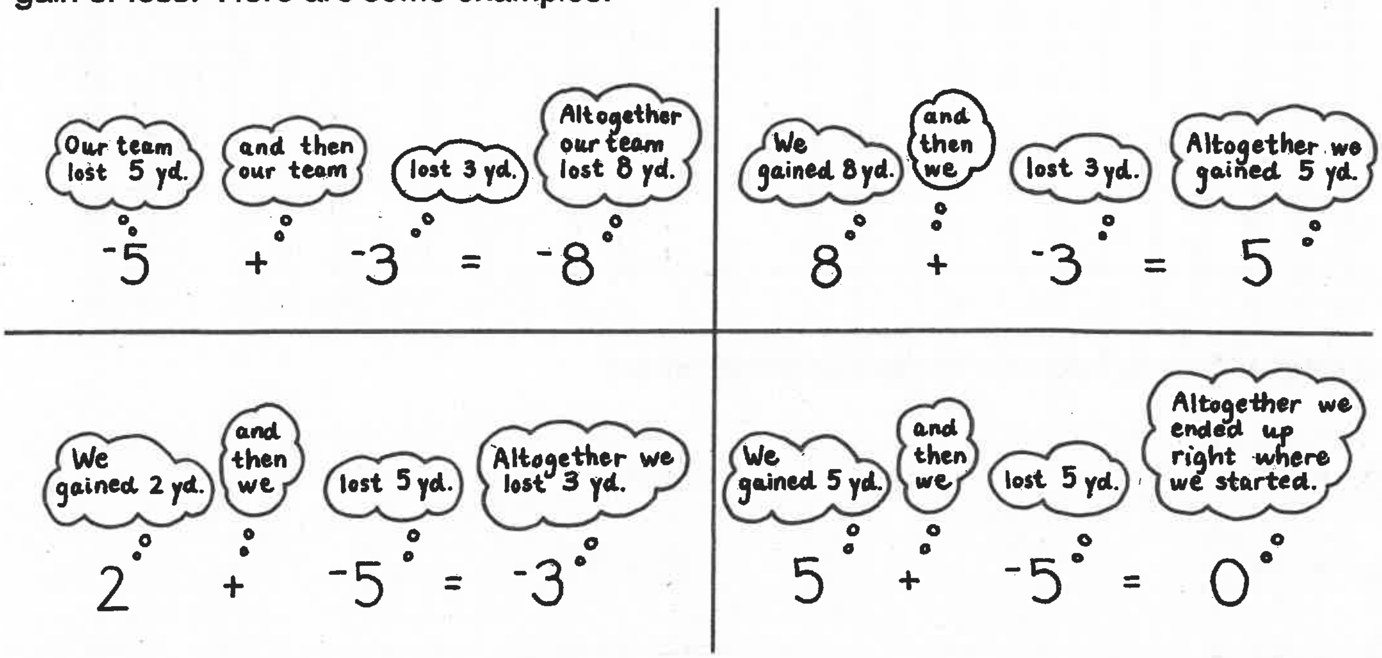
$^-4$ is less than $^-2$.

1	>	$^-2$	$^-4$	<	$^-2$	$^-2$	$^-4$
5		2	$^-7$		$^-3$	2	$^-4$
2		5	$^-4$		$^-4$	$^-5$	5
$^-2$		0	3		$^-3$	1	$^-1$
0		2	$^-2$		$^-6$	$^-1$	0

Adding Integers

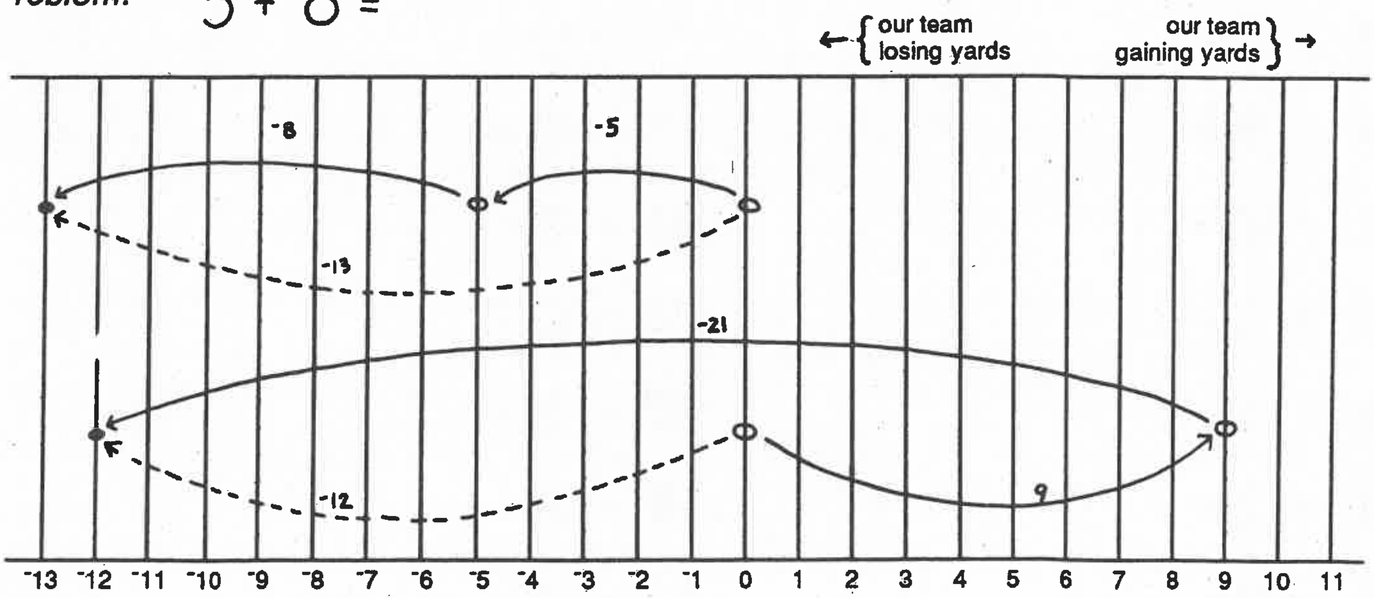
In arithmetic you learned the operations of adding, subtracting, multiplying and dividing whole numbers and fractions. In algebra, one of the first things you have to learn is how to add, subtract, multiply and divide integers.

To add integers we can think of a football game. A positive number stands for ground gained by our team; a negative number shows ground lost. Zero is used when there is no gain or loss. Here are some examples:

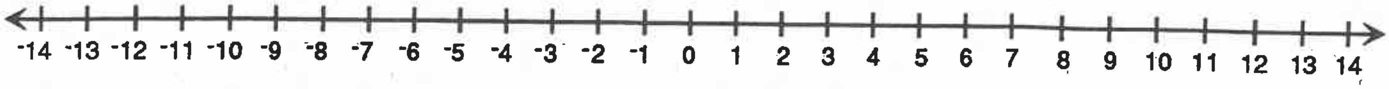


If you ever have trouble adding integers, then you can draw a football field to help you figure out the answer.

Problem: $^{-}5 + {}^{-}8 =$



Problem: $9 + {}^{-}21 =$



Add.

$^{-}6 + {}^{-}3 =$	$4 + 5 =$	$3 + 2 =$	$7 + 6 =$
$8 + {}^{-}5 =$	$^{-}9 + {}^{-}2 =$	$3 + 1 =$	$8 + {}^{-}4 =$
$^{-}5 + {}^{-}3 =$	$^{-}5 + {}^{-}8 =$	$3 + 0 =$	$9 + {}^{-}7 =$
$^{-}8 + 8 =$	$^{-}9 + 4 =$	$3 + {}^{-}1 =$	$^{-}7 + {}^{-}7 =$
$^{-}6 + 5 =$	$10 + {}^{-}8 =$	$3 + {}^{-}2 =$	$^{-}7 + 9 =$
$12 + {}^{-}6 =$	$2 + {}^{-}11 =$	$3 + {}^{-}3 =$	$^{-}11 + 5 =$
$^{-}11 + 0 =$	$13 + {}^{-}9 =$	$3 + {}^{-}4 =$	$^{-}1 + 1 =$
$^{-}16 + 7 =$	$^{-}13 + 5 =$	$3 + {}^{-}5 =$	$17 + {}^{-}8 =$
$8 + {}^{-}3 =$	$^{-}5 + 9 =$	$3 + {}^{-}6 =$	$^{-}14 + 14 =$

2	$^{-}5$	6	$^{-}7$	9	$^{-}11$	$^{-}12$	$^{-}11$
$+\ 6$	$+\ {}^{-}6$	$+\ {}^{-}9$	$+\ {}^{-}4$	$+\ {}^{-}3$	$+\ 4$	$+\ 15$	$+\ 11$

$14 + 0 =$	$^{-}6 + 0 =$	$37 + 0 =$	$^{-}65 + 0 =$
$0 + 3 =$	$0 + {}^{-}8 =$	$0 + {}^{-}15 =$	$0 + 39 =$

Adding zero is easy! We just have to look at the other number in the problem and that's the answer. Here is a way we can say this:

> If a is any integer, then $a + 0 = a$
> and $0 + a = a$.

This is called the Principle for Adding Zero.

The problems on this page are too hard . . .

Make them easier by finding opposites and getting rid of them.

$(8 + {}^-8 = 0)$

$\cancel{8} + 6 + {}^-\cancel{8} + 4 = 10$

$5 + 2 + {}^-5 + 6 =$

${}^-7 + 6 + {}^-4 + {}^-6 =$

$9 + {}^-2 + 8 + {}^-9 =$

$3 + {}^-8 + {}^-3 + 8 =$

$37 + 4 + {}^-37 + {}^-5 =$

$6 + {}^-9 + 8 + 9 + {}^-6 + 3 =$

${}^-8 + 8 + 5 + 9 + {}^-5 + 6 + {}^-9 =$

$17 + {}^-28 + 56 + 28 + {}^-17 =$

${}^-12 + 5 + {}^-6 + 12 + {}^-5 =$

James is still playing with his friends. He won 25 points. Then he lost 17, won 2, lost 19, won 2, won 17, lost 33, won 19, lost 25 and won 33. How did he come out?

Problem:

Answer:

In this subtraction problem the number being subtracted is negative:

$$7 - {}^{-}5 =$$

First we have to change the problem. We have to add the opposite of the second number, so instead of subtracting negative 5 we are going to add positive 5:

$$7 + {}^{+}5 =$$

A gain of 7 and a gain of 5 is the same as a gain of 12:

$$7 + {}^{+}5 = 12$$

If this seems strange to you, think of the football field. When the referee takes away or rules out a 5 yard loss, we gain back the 5 yards.

Subtract.

${}^{-}3 + {}^{+}7 = 4$

$5 - {}^{-}9 =$

$8 - {}^{-}3 =$

${}^{-}6 - {}^{-}4 =$

${}^{-}9 - {}^{-}6 =$

${}^{-}8 - {}^{-}8 =$

$10 - {}^{-}5 =$

${}^{-}10 - {}^{-}5 =$

$2 - {}^{-}9 =$

${}^{-}2 - {}^{-}9 =$

Be careful on these.

$6 - {}^{-}8 =$

$6 - 8 =$

${}^{-}6 - {}^{-}8 =$

${}^{-}6 - 8 =$

As you can see, every time we have a subtraction problem, we can change it to an adding problem. But we have to remember to *add the opposite* of the second number:

If a and b are integers, then $a - b = a + -b$.

Here are some longer problems.

13 - 3 - 6 - 1 =

16 - 10 - 2 - 3 =

8 - 2 - 2 - 2 =

10 - 3 - 3 - 4 =

9 - 3 - 6 - 5 =

12 - 7 - 5 - 4 =

8 - 5 - 5 - 5 =

6 - 5 - 4 - 3 =

11 - 9 - 6 - 6 =

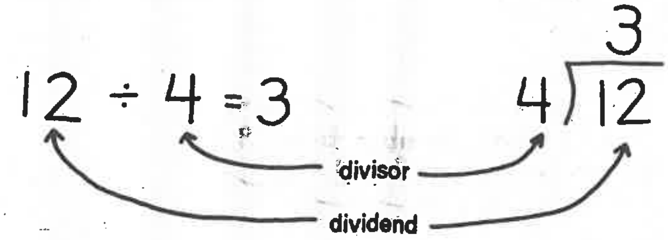
3 - 7 - 4 - 2 =

12 - 15 - 10 - 8 =

7 - 7 - 7 - 7 =

Dividing Integers

In division problems, the number we divide *by* is called the **divisor** and the number we divide *into* is called the **dividend**.



When you divide integers you have to break down the problem into two parts. We find the *amount* of the answer by dividing, and we find the *sign* of the answer by following these rules:

POSITIVE	•	POSITIVE	=	POSITIVE
POSITIVE	•	NEGATIVE	=	NEGATIVE
NEGATIVE	•	POSITIVE	=	NEGATIVE
NEGATIVE	•	NEGATIVE	=	POSITIVE

These rules are the same as the rules for multiplication. That's because the answer to a division problem can be found by reversing a multiplication problem.

$18 \div 3 = 6$	because	6	times	3	is	18.
$18 \div -3 = -6$	because	-6	times	-3	is	18.
$-18 \div 3 = -6$	because	-6	times	3	is	-18.
$-18 \div -3 = 6$	because	6	times	-3	is	-18.

Here are some division problems for you:

$12 \div 3 =$ 4	$10 \div 5 =$	$-18 \div 6 =$
$-12 \div -3 =$ 4	$-10 \div -5 =$	$18 \div 6 =$
$12 \div -3 =$ -4	$10 \div -5 =$	$18 \div -6 =$
$-12 \div 3 =$ -4	$-10 \div 5 =$	$-18 \div -6 =$
$-7 \div -7 =$	$-15 \div 1 =$	$-9 \div -9 =$
$-7 \div 7 =$	$-15 \div -1 =$	$-9 \div 9 =$
$7 \div 7 =$	$15 \div -1 =$	$0 \div 9 =$
$7 \div -7 =$	$15 \div 1 =$	$9 \div 0 =$

Did you get the last two problems? If not, the next page may help you.

Order of Operations

Parentheses
Exponents
Multiplication
Division
Addition
Subtraction

(Left to right whatever comes first)

We saw that parentheses are often used in algebra to show what to do first. Look at this problem:

$$4 \cdot (5 + 2) =$$

The parentheses tell us to add 5 + 2 first, and then to multiply the answer by 4:

$$4 \cdot \underbrace{(5 + 2)}_{7} = 28$$

Below are some problems for you to do.

$5 \cdot (2 + 3) =$ $\quad$ $7 - (5 - 2) =$ $\quad$ $(^{-}6 + 4) \cdot 3 =$

$(5 \cdot 2) + 3 =$ $\quad$ $(7 - 5) - 2 =$ $\quad$ $^{-}6 + (4 \cdot 3) =$

$5 + (2 \cdot 3) =$ $\quad$ $7 + (5 + 2) =$ $\quad$ $(^{-}2 \cdot {}^{-}2) + 6 =$

$(5 + 2) \cdot 3 =$ $\quad$ $(7 + 5) + 2 =$ $\quad$ $^{-}2 \cdot (^{-}2 + 6) =$

$(4 - 3) \cdot (5 - 1) =$ $\quad$ $(3 \cdot 4) + (4 \cdot 2) + (3 \cdot 3) =$

$(3 - 5) \cdot (10 - 6) =$ $\quad$ $(^{-}3 \cdot 4) + (4 \cdot 2) + (^{-}3 \cdot {}^{-}3) =$

$(5 - 3) \cdot (6 - 10) =$ $\quad$ $(^{-}3 \cdot {}^{-}4) + (^{-}4 \cdot 2) + (3 \cdot {}^{-}3) =$

$(3 - 5) \cdot (6 - 10) =$ $\quad$ $(^{-}3 \cdot {}^{-}4) + (^{-}4 \cdot {}^{-}2) + (^{-}3 \cdot 3) =$

$(5 - 3) \cdot (10 - 6) =$ $\quad$ $(^{-}3 \cdot 4) + (4 \cdot {}^{-}2) + (^{-}3 \cdot 3) =$

Keep denominator and add numerator.

Ex: $\frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$

Adding Simple Fractions (common denominator)

1) $\frac{1}{10} + \frac{1}{10} =$

2) $\frac{2}{10} + \frac{4}{10} =$

3) $\frac{2}{9} + \frac{2}{9} =$

4) $\frac{2}{11} + \frac{4}{11} =$

5) $\frac{1}{6} + \frac{4}{6} =$

6) $\frac{2}{9} + \frac{6}{9} =$

7) $\frac{1}{3} + \frac{1}{3} =$

8) $\frac{1}{4} + \frac{1}{4} =$

9) $\frac{2}{8} + \frac{3}{8} =$

10) $\frac{1}{7} + \frac{3}{7} =$

Find a common denominator, then keep the denominator and subtract the numerators.

EX: $\frac{8}{10} - \frac{1}{2} = \frac{8}{10} - \frac{1 \cdot 5}{2 \cdot 5} = \frac{8}{10} - \frac{5}{10} = \frac{3}{10}$

Subtracting Fractions

1) $\frac{9}{10} - \frac{1}{2} =$

2) $\frac{3}{4} - \frac{1}{2} =$

3) $\frac{3}{5} - \frac{1}{3} =$

4) $\frac{3}{4} - \frac{1}{10} =$

5) $\frac{2}{5} - \frac{1}{4} =$

6) $\frac{1}{2} - \frac{1}{3} =$

7) $\frac{8}{10} - \frac{2}{4} =$

8) $\frac{8}{10} - \frac{2}{3} =$

9) $\frac{1}{2} - \frac{1}{5} =$

10) $\frac{4}{10} - \frac{1}{3} =$

Instead of dividing by a fraction, multiply by the reciprocal (flip second fraction upside down)

EX: $\frac{3}{4} \div \frac{1}{2} = \frac{3}{4} \times \frac{2}{1} = \frac{6}{4} = 1\frac{2}{4} = 1\frac{1}{2}$

Dividing Fractions

1) $\frac{1}{2} \div \frac{1}{4} =$

2) $\frac{6}{10} \div \frac{4}{5} =$

3) $\frac{3}{4} \div \frac{7}{10} =$

4) $\frac{4}{10} \div \frac{2}{4} =$

5) $\frac{7}{10} \div \frac{1}{2} =$

6) $\frac{3}{5} \div \frac{1}{2} =$

7) $\frac{2}{3} \div \frac{1}{4} =$

8) $\frac{2}{5} \div \frac{1}{3} =$

9) $\frac{5}{10} \div \frac{3}{4} =$

10) $\frac{1}{4} \div \frac{1}{2} =$